Asset Allocation Based On The Filtered Historical Simulation Value-at-Risk*

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Abstract

The paper proposes the Filtered Historical Simulation (FHS) Value-at-Risk computed at 95% and 99%, for ranking a certain number of stocks. FHS retains the non-parametric nature of historical simulation by bootstrapping from the standardized residuals. These bootstrapped standardized residuals are then used to generate time paths of future asset returns. The empirical analysis is based on the constituents of the Dow Jones Industrial Average index and derives the results for a simple exercise of asset allocation. The simulations assess the Value-at-Risk, relying on 50,000 and 1,000,000 independent random trials over n. 10 and n. 22 trading days.

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1. Introduction

Individual investors tend to rely on the naive diversification strategy rather than sophisticated diversification to allocate their wealth across assets. The naive 1/N diversification rule is the strategy in which a fraction 1/N of wealth is allocated to each of the N assets available for investment at each rebalancing date. For example, Benartzi and Thaler (2001) and Liang and Weisbenner (2002) find that investors follow the naive 1/N strategy to allocate their wealth across assets. The naive strategy does not align with the mean–variance framework of optimal asset allocation strategy, which suggests giving more weight to those assets that contribute to higher mean–variance efficiency (Markowitz 1952, Litterman 2003, Campbell and Viceira 2002, Brandt 2010).

The most appealing feature of the 1/N portfolio is that it is easy to implement since it does not require any estimation of the moments of asset returns, optimization, and short sales. Previous literature documents that optimal portfolio strategy does not dominate the naive 1/N strategy. For instance, Bloomfield, Leftwich, and Long (1977) show that sample-based mean–variance optimal strategies do not outperform a simpler strategy of maintaining equal dollar investments in each available stock.

This paper proposes a simple exercise of asset allocation based on the equal partition of capital to the constituents of the Dow Jones Industrial Average (DJIA) index, with the aim to create portfolios of stocks. The exercise ranks the constituents of the DJIA index, by the derived Filtered Historical Simulation (FHS) Value-at-Risk (VaR) values computed at 95% and 99% percentiles, considering n. 50,000 and n. 1,000,000 independent random trials, over n. 10 trading days and a one month horizon of n. 22 trading days. It assembles them in sextiles and tertiles, with the aim to compute the average rates of return and the cumulative rates of return of the formed portfolios. The first sextile (tertile) of the constituents reports the highest market risk, i.e. the lowest value of the FHS VaR; whereas, the last sextile (tertile) of the constituents reports the lowest market risk, i.e. the highest value of the FHS VaR.

The first portfolios, characterized by stocks of the DJIA index with the lowest values of the FHS VaR, computed at the percentile levels of 95% and 99%, realize the highest rates of return; whereas, the last portfolios, characterized by stocks of the DJIA index with the highest values of the FHS VaR, report the lowest performance. The estimated results are impacted by outliers that bias the computations of the rates of return for the portfolios. In particular, the average rate of return for the fourth sextile characterized by a medium level of market risk reports the highest performance.

The empirical analysis also reports the cumulative rates of return for the formed portfolios. The first portfolios, characterized by stocks of the DJIA index with the lowest values of the FHS VaR, are more risky and so report greater cumulative rates of return; whereas, the last portfolios, characterized by stocks of the DJIA index with the highest values of the FHS VaR, are less risky and provide the lowest cumulative rates of return.

The rest of the paper is organized as follows: Section 2 provides an overview of the FHS VaR. Section 3 discusses the data and delivers the descriptive statistics. Section 4 proposes the econometric methodology. Section 5 reports the empirical results; whereas, Section 6 concludes the paper.

2. The Filtered Historical Simulation: An Overview

The essence of the filtered historical simulation (FHS) idea is to use the information in recent observed returns to estimate the current level of risk more accurately. The FHS technique was first introduced with the aim to address the issue that percentiles in the tail of observed return distributions were neither well characterized by simple functional forms such as the normal distribution nor sometimes by the returns in the models' data windows. If the conditional volatility changes, then the statistical procedure needs to update the Value-at-Risk estimates. In FHS, the historical returns are first standardized by the volatility estimated on that particular day, so the stationarity assumption is relaxed. This filtering process yields approximately independent and identically distributed (i.i.d) returns (residuals) suited for historical simulation. The observed returns are scaled by the current conditional forecast of volatility and then used as innovations, reflecting also the current market conditions. FHS retains the non-parametric nature of historical simulation by bootstrapping from the standardized residuals. These bootstrapped standardized residuals are then used to generate time paths of future asset returns. The "unfiltered" historical simulation is a special case of FHS, which holds when returns are i.i.d.

A major advantage of FHS over historical simulation (HS) is that the filtering process increases the range of outcomes beyond the historical record through a change of scale. In other words, FHS provides a systematic approach to generate extreme events not present in the historical record, completing the tails of the distribution. Therefore, FHS requires a shorter historical record than HS to simulate the tails of the distribution of returns. FHS is a generalized historical simulation and overcomes most of the historical simulation weaknesses.

Barone-Adesi et al. (2002) point out how the FHS technique can be adapted to stress testing because it simulates the whole distribution of security returns. The technique samples from more extreme points in the tails of the multivariate distribution by increasing the number of simulation runs. FHS can produce risk measures that are consistent with the current state of markets at any arbitrarily large confidence level. These impressive gains allow FHS to dominate historical simulation (HS) easily. Indeed, HS fails to condition forecasts on the current state of the markets. The major attraction of FHS is that it does not rely on any assumptions about the distribution of returns.

3. Data and Descriptive Statistics

The empirical analysis is based on the constituents of the Dow Jones Industrial Average (DJIA) index, that is a stock market index of 30 prominent companies listed on stock exchanges in the United States. It is a price-weighted measure of 30 U.S. blue-chip companies.

The index covers all industries except transportation and utilities. The value of the index can also be calculated as the sum of the stock prices of the companies included in the index, divided by a factor, which is approximately 0.152 as of April 2024. The factor is changed whenever a constituent company undergoes a stock split so that the value of the index is unaffected by the stock split. Table 1 reports the summary and descriptive statistics for the constituents of the index from January 1st, 1973 to July 30th, 2024.

[Please Insert Table 1 around here]

The median value for the stock prices ranges from 10.844 for the company Walmart to 163.445 for the institution Goldman Sachs. The average value of the median values across stocks is equal to 37.215; whereas, the standard deviation ranges from 7.79 for the company Dow Inc. to 141.605 for the institution United Health. The average value of the standard deviations across the constituents is equal to 53.371.

4. The Econometric methodology

Under the historical measure **P**, the stock returns are modeled with an asymmetric Exponential GARCH(1,1) specification (Nelson 1991) and an empirical innovation density generated with the Filtered Historical Simulation technique (Barone-Adesi et al. 1999). To produce series of i.i.d. observations, the procedure fits a first order autoregressive model to the conditional mean of the stock returns (r_t):

$$r_t = c + \theta \cdot r_{t-1} + \epsilon_t, \tag{1}$$

where, c is the constant of the mean equation and θ is the coefficient of the autoregressive component. The conditional variance follows an asymmetric exponential GARCH (EGARCH) specification,

$$log\left[\sigma_t^2\right] = k + \alpha \cdot log\left[\sigma_{t-1}^2\right] + \phi\left(|z_{t-1}| - E\left[|z_{t-1}|\right]\right) + \psi \cdot z_{t-1}, \quad (2)$$

where, k, α, $\phi\, and\, \psi$ are the coefficients of the conditional variance process. In particular, k is the coefficient that depicts the long run variance; α is the coefficient related to the persistence of the variance component; ϕ is the coefficient that depicts the deviation of the lagged innovations from the expected value; whereas, ψ is the unknown parameter that accounts for the asymmetry effect.

Additionally, the standardized residuals of the constituents are modeled as standardized Student's t distributions to compensate for the fat tails often associated with equity returns. That is,

$$z_t = \epsilon_t / \sigma_t. \quad (3)$$

The standardized residuals are now approximately independent and identically distributed (i.i.d.) and far more amenable to subsequent stationary bootstrapping. If $\phi > 0$, a deviation of $|z_{t-1}|$ from its expected value $E\left[|z_{t-1}|\right]$ implies the variance of ϵ_t to be larger than otherwise. The term ψ, if smaller than zero, accounts for the asymmetry effect, i.e. negative surprises ($z_{t-1} < 0$) raise the future stock returns volatility more than positive surprises ($z_{t-1} \geq 0$) of the same absolute magnitude.

5. Empirical Results

This section discusses the estimates and the empirical results of the econometric methodology proposed in Section 4 for modeling the constituents of the DJIA index and applying the FHS.

[Please Insert Table 2 around here]

Table 2 contains the estimated values of the AR(1)-EGARCH(1,1) processes with t-student distribution of the innovations. The coefficients of the conditional variances for the constituents of the DJIA index are almost all statistically significant, with exception of the unknown parameter θ that for some constituents is not statistically significant, implying that the autoregressive component of the mean equation that describes the evolution of the constituents is statistically negligible.

The estimation of the coefficients for the AR(1)-EGARCH(1,1) processes relies on the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm (Roger 1987) that is an iterative method for solving unconstrained nonlinear optimization problems. It belongs to quasi-Newton methods and seeks a stationary point of a function, reachable when the gradient is zero. The optimization algorithm begins at an initial estimate for the optimal values and proceeds iteratively to get better estimates at each stage, till when there is a convergence for finding the solutions. For simplicity, the maximum number of iterations is fixed to n. 5,000 and the convergence rate to 1e-06. The step method is based on the Levenberg-Marquardt algorithm (Levenberg 1944; Marquardt 1963) that is more robust than the Gauss-Newton algorithm, since it allows to derive solutions even if the algorithm starts very far o from the final minimum. In cases with multiple minima, the algorithm converges to the global minimum only if the initial guess is already somewhat close to the final solution. The estimation procedure also accommodates the Huber-White estimator (Huber 1967; White 1980), that allows to derive the variance/covariance matrix considering the heteroscedasticity of the residuals.

The estimation of the conditional variance processes allows to derive the standardized residuals and so apply the bootstrapping procedure that is the key step for computing the Value-at-Risk (VaR) based on the FHS. It produces independent and identically distributed (i.i.d) standardized residuals consistent with those obtained from the AR(1)-EGARCH(1,1) filtering. The i.i.d property is important for bootstrapping and allows the sampling procedure to safely avoid the pitfalls of sampling from a population in which successive observations are serially dependent.

[Please Insert Table 3 around here]

Table 3 contains the derived results of the FHS VaR computed at 95% and 99% and reports the values of the FHS computed for the constituents of the DJIA index, simulating n. 50,000 (Panel 3.1) independent random trials of standardized residuals over n. 10 trading days and a one month horizon of n. 22 trading days. The simulating exercise is also performed for n. 1,000,000 (Panel 3.2) independent random trials, with the aim to test the adequacy of the bootstrapping procedure and so the reliability of the FHS VaR values.

Panel 3.1 and Panel 3.2 show the stability of the bootstrapping technique since the FHS VaR values are almost the same, for n. 50,000 and n. 1,000,000 independent random trials.

5.1 Asset Allocation: A Study For The DJIA Index

This sub-section proposes a simple exercise of asset allocation based on the equal partition of capital to the constituents of the DJIA index, with the aim to create portfolios of stocks. The exercise ranks the constituents by the derived FHS VaR values computed at 95% and 99% percentiles, considering n. 50,000 and n. 1,000,000 independent random trials, over n. 10 trading days and a one month horizon of n. 22 trading days. It assembles them in sextiles and tertiles, with the aim to compute the average rates of return and the cumulative rates of return of the formed portfolios. The first sextile (tertile) of the constituents reports the highest market risk, i.e. the lowest value of the FHS VaR; whereas, the last sextile (tertile) of the constituents reports the lowest market risk, i.e. the highest value of the FHS VaR.

[Please Insert Table 4 around here]

Before to start the analysis, Table 4 contains the prices and the realized rates of return for the 30 constituents of the DJIA index, computed as percentage variations of the stock prices at one week and one month, from June 28th, 2024. For a week horizon period, the best performer is APPLE that realizes 7.46%. The market performance of this stock reduces to 3.48%, for the period of one month. The worst performer stock is UNITED HEALTH that only realizes -4.17%. The realized rate of return increases to 11.87%, for the period of one month. It is also interesting to note the performance of the stock 3M that for one week horizon realizes -0.85% and increases to 24.43% in one month horizon.

[Please Insert Table 5 around here]

Table 5 reports the results of the stock performance for the simple exercise of asset allocation. In particular, Panel 5.1 contains the average rates of return at one week and one month, for the created portfolios, based on sextiles of the constituents; whereas, Panel 5.2 shows the cumulative rates of return for the formed portfolios.

In line with the conventional wisdom, the first portfolios, characterized by stocks of the DJIA index with the lowest values of the FHS VaR, computed at the percentile levels of 95% and 99%, realize the highest rates of return; whereas, the last portfolios, characterized by stocks of the DJIA index with the highest values of the FHS VaR, report the lowest performance. The estimated results are impacted by outliers that bias the computations of the rates of return for the portfolios. In particular, the average rate of return for the fourth sextile characterized by a medium level of market risk reports the highest performance. Further, Panel 5.1 also reveals how the results of the performance are consistent if an investor holds the portfolios for one week and one month horizon.

The cumulative rates of return for the formed portfolios are shown in Panel 5.2. The first portfolios, characterized by stocks of the DJIA index with the lowest values of the FHS VaR, are more risky and so report greater cumulative rates of return; whereas, the last portfolios, characterized by stocks of the DJIA index with the highest values of the FHS VaR, are less risky and provide the lowest cumulative rates of return[1].

[Please Insert Table 6 around here]

Table 6 shows the findings of the analysis for the portfolios formed on the tertiles of the constituents. They are more consistent and in line with the expectations, since the performance of the constituents that are outliers is mitigated with the computations. The first portfolio, characterized by stocks of the DJIA index with the lowest values of the FHS VaR realizes the highest rates of return; whereas, the last portfolio, characterized by stocks of the DJIA index with the highest values of the FHS VaR, report the lowest performance.

6. Conclusion

The aim of this paper is to propose a simple asset allocation strategy, based on the ranking of stocks by the FHS VaR at the 95% and 99% confidence levels, using both 50,000 and 1,000,000 simulated paths, and the equal partition of capital to the constituents of the DJIA index, with the aim to create portfolios of stocks.

The empirical findings clearly indicate a relationship between market risk (as measured by FHS VaR) and portfolio returns. In general, the portfolios composed of stocks with lower VaR values achieve higher average and cumulative returns, both over one-week and one-month investment time horizons. This pattern was observed across both sextiles and tertiles groupings of stocks. The results also highlight the presence of outliers that may distort returns within some risk buckets. Nonetheless, the results based on tertiles partitions were more stable and in line with the theoretical expectations, thus suggesting that a greater granularity may help to mitigate the influence of extreme values.

[1] There are outliers in the formed portfolios that jeopardize the computation of the cumulative rates of return. The fourth portfolio reports a cumulative rate of return above 40%, considering an investor that holds it for one month.

The results reaffirm the usefulness of FHS-based risk metrics in asset allocation decisions. Indeed, while the naive 1/N diversification rule is simple, the incorporation of risk-adjusted measures (such as FHS VaR) can lead to the building of more informative portfolios.

The empirical analysis also shows that riskier portfolios (defined by lower simulated VaR) tend to reward investors with higher returns, thus providing a basis for ranking stocks not only on return expectations, but also on conditional risk exposures. Furthermore, the stability of FHS results across simulation scales (50,000 vs. 1,000,000 draws) is also interesting because it suggests that the proposed technique is both robust and computationally scalable.

The research findings discussed in the paper could be extended in various directions for future studies. Indeed, extending the analysis to other asset classes (such as bonds or commodities) or market regimes (such as crisis vs. expansion), could test the generalizability of a FHS-based allocation strategy. In this respect, an interesting line of research could involve the incorporation of transaction costs and rebalancing frequency, while introducing realistic frictions that could allow a more accurate assessment of the strategy's net performance. A second possible line of future research could involve a comparison of the proposed method with alternative risk measures, (such as, for instance, Expected Shortfall or Entropic VaR), in order to stress its relative advantages and limitations. Furthermore, incorporating data-driven techniques (such as clustering, classification or neural networks) into the ranking and allocation process might improve outlier detection and portfolio performance under non-stationary conditions. Finally, enhancing the FHS framework to simulate macroeconomic or geopolitical shocks could make it more useful for regulatory and risk management purposes.

In conclusion, the FHS VaR approach provides a flexible and effective tool for risk-sensitive asset allocation, with potential for both academic developments and practical implementations.

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Table 1.
Summary and Descriptive Statistics

The table reports the descriptive statistics (mean, median, max., min. and standard deviation) for the constituents of the Dow Jones Industrial Average (DJIA), that is a stock market index of 30 prominent companies listed on stock exchanges in the United States. The value of the index can also be computed as the sum of the stock prices of the companies included in the index, divided by a factor, which is approximately 0.152 as of April 2024. The factor is changed whenever a constituent company undergoes a stock split so that the value of the index is unaffected by the stock split. The descriptive statistics are from January 1st, 1973 to July 30th, 2024.

	Company Name	Descriptive Statistics					
		Mean	**Median**	**Max.**	**Min.**	**Standard Deviation**	**N. observations**
1	**3M**	52.578	36.761	216.234	4.351	49.961	13434
2	**AMAZON**	36.873	8.498	197.850	0.070	53.120	7077
3	**AMERICAN EXPRESS**	40.481	24.656	243.080	1.136	46.850	13434
4	**AMGEN**	76.037	56.075	324.560	0.078	81.097	10706
5	**APPLE**	21.982	0.529	216.670	0.049	45.295	11361
6	**BOEING**	68.527	38.813	440.620	0.383	85.317	13434
7	**CATERPILLAR**	51.623	21.880	379.300	3.365	65.411	13434
8	**CHEVRON**	51.273	37.176	188.050	2.547	46.107	13434
9	**CISCO**	23.968	21.670	80.063	0.071	16.952	8966
10	**COCA-COLA**	21.543	21.925	66.210	0.473	18.771	13434
11	**DISNEY**	36.324	21.510	201.910	0.334	43.342	13434
12	**DOW**	53.399	53.830	70.910	22.000	7.793	1378
13	**GOLDMAN SACHS**	177.793	163.445	470.410	52.000	87.882	6564
14	**HOME DEPOT**	68.233	33.060	416.180	0.035	95.521	11159
15	**HONEY WELL**	64.014	39.290	234.180	5.980	60.724	10117

(continue)

	Company Name	Descriptive Statistics					
		Mean	**Median**	**Max.**	**Min.**	**Standard Deviation**	**N. observations**
16	**IBM**	73.750	61.175	206.124	9.074	56.654	13434
17	**INTEL**	17.511	17.896	74.875	0.029	17.696	13434
18	**JOHNSON & JOHNSON**	49.396	38.234	186.010	1.305	51.400	13434
19	**JPMORGAN CHASE**	40.099	31.536	204.790	3.375	40.937	13434
20	**MCDONALD'S**	56.123	24.313	300.530	0.525	74.888	13434
21	**MERCK**	33.657	30.599	132.960	1.206	29.264	13434
22	**MICROSOFT**	57.692	27.330	452.850	0.090	88.527	9992
23	**NIKE**	26.087	7.313	177.510	0.107	38.347	11369
24	**PROCTER & GAMBLE**	26.020	22.625	71.560	3.800	14.270	13434
25	**SALESFORCE**	86.759	57.030	316.880	2.398	82.197	5223
26	**TRAVELERS**	48.183	33.000	230.890	1.922	50.243	13434
27	**UNITED HEALTH**	89.400	26.420	551.150	0.086	141.605	10347
28	**VERIZON**	34.895	34.071	62.282	7.295	13.861	10595
29	**VISA**	106.961	78.270	290.370	10.605	82.434	4248
30	**WALMART**	13.695	10.844	68.900	0.005	14.661	13434

Table 2.
Estimation Results

The table contains the estimated values of the **AR(1)-EGARCH(1,1)** with t-student distribution of the innovations. c is the constant of the mean equation multiplied by 1000; θ is the coefficient that depicts the autoregressive component; k is the coefficient that depicts the long run variance; α is the coefficient that depicts the persistence of the variance component; ϕ is the coefficient that depicts the deviation of the lagged innovations from the expected value; ψ is the coefficient that accounts for the asymmetry effect; t is the number related to the degrees of freedom for the t-student distribution of the innovations. The data are from January 1st, 1973 to July 30th, 2024. *, **, *** respectively represent the statistical level of significance at 10%, 5% and 1%.

Company Name	**Coefficients**						
	c	θ	k	α	ϕ	ψ	t
3M	0.241***	-0.001	-0.071***	0.992***	0.097***	-0.032***	4.372***
AMAZON	0.853***	0.006	-0.020***	0.997***	0.113***	-0.031***	3.835***
AMERICAN EXPRESS	0.329***	0.004	-0.063***	0.992***	0.126***	-0.056***	5.573***
AMGEN	0.294**	-0.001	0.048***	0.994***	0.136***	-0.033***	4.432***
APPLE	0.910***	0.011	-0.070***	0.991***	0.135***	-0.029***	4.480***
BOEING	0.381***	0.011	-0.090***	0.989***	0.122***	-0.037***	5.044***
CATERPILLAR	0.214*	0.048***	-0.066***	0.992***	0.093***	-0.037***	4.729***
CHEVRON	0.328***	0.016*	-0.102***	0.988***	0.124***	-0.036***	6.076***
CISCO	0.722***	-0.008	-0.024***	0.997***	0.110***	-0.033***	4.615***
COCA-COLA	0.282***	0.010	-0.064***	0.993***	0.119***	-0.033***	5.052***
DISNEY	0.326***	0.018**	-0.066***	0.992***	0.109***	-0.030***	4.943***
DOW	-0.348	0.016	-0.041**	0.995***	0.062***	-0.069***	6.682***
GOLDMAN SACHS	0.453**	-0.019	-0.081***	0.990***	0.125***	-0.054***	5.770***
HOME DEPOT	0.711***	0.032***	-0.074***	0.991***	0.142***	-0.043***	4.751***
HONEY WELL	0.360***	-0.019**	-0.107***	0.987***	0.131***	-0.071***	5.008***

(continue)

Company Name	**Coefficients**						
	c	θ	k	α	ϕ	ψ	t
IBM	0.113	-0.025***	0.209***	0.992***	0.112***	-0.031***	4.622***
INTEL	0.639***	0.022***	-0.040***	0.995***	0.107***	-0.022***	5.125***
JOHNSON & JOHNSON	0.224***	0.023***	-0.109***	0.988***	0.148***	-0.052***	5.610***
JPMORGAN CHASE	0.276**	0.013	-0.075***	0.991***	0.138***	-0.051***	5.231***
MCDONALD'S	0.446***	0.006	-0.058***	0.993***	0.110***	-0.031***	5.449***
MERCK	0.227**	0.027***	-0.124***	0.985***	0.107***	-0.040***	4.952***
MICROSOFT	0.721***	-0.021**	-0.089***	0.989***	0.161***	-0.029***	4.772***
NIKE	0.388***	0.009	-0.055***	0.993***	0.112***	-0.042***	3.919***
PROCTER & GAMBLE	0.364***	-0.039***	-0.087***	0.990***	0.127***	-0.025***	4.446***
SALESFORCE	0.874***	-0.034***	-0.063***	0.992***	0.109***	-0.058***	3.979***
TRAVELERS	0.359***	0.035***	-0.101***	0.988***	0.151***	-0.047***	4.021***
UNITED HEALTH	0.735***	-0.001	-0.062***	0.992***	0.137***	-0.048***	3.968***
VERIZON	0.251**	-0.011	-0.075***	0.991***	0.120***	-0.027***	5.112***
VISA	0.978***	-0.071***	-0.146***	0.983***	0.169***	-0.090***	4.911***
WALMART	0.434***	0.005	-0.040***	0.995***	0.119***	-0.020***	4.748***

Table 3.
Filtered Historical Simulation Value-at-Risk

The table contains the results of the Filtered Historical Simulation (**FHS**) Value-at-Risk computed at 95% and 99%. The bootstrapping procedure produces independent and identically distributed (i.i.d) standardized residuals consistent with those obtained from the **AR(1)-EGARCH(1,1)** filtering. The table reports the values of the **FHS** computed for the constituents of the Dow Jones Industrial Average (DJIA), that is a stock market index of 30 prominent companies listed on stock exchanges in the United States, simulating n. 50,000 (**Panel 3.1**) and n. 1,000,000 (**Panel 3.2**) independent random trials of standardized residuals over n. 10 trading days and one month horizon of n. 22 trading days.

Panel 3.1: FHS Value-at-Risk with n. 50,000 independent random trials

Company Name	FHS Value-at-Risk				Company Name	FHS Value-at-Risk			
	95%		99%			95%		99%	
	10 days	22 days	10 days	22 days		10 days	22 days	10 days	22 days
3M	-0.063	-0.095	-0.102	-0.157	**IBM**	-0.058	-0.088	-0.094	-0.144
AMAZON	-0.078	-0.112	-0.128	-0.192	**INTEL**	-0.092	-0.138	-0.153	-0.224
AMERICAN EXPRESS	-0.061	-0.093	-0.097	-0.154	**JOHNSON & JOHNSON**	-0.049	-0.075	-0.079	-0.124
AMGEN	-0.070	-0.107	-0.113	-0.173	**JPMORGAN CHASE**	-0.068	-0.105	-0.110	-0.172
APPLE	-0.084	-0.125	-0.140	-0.209	**MCDONALD'S**	-0.060	-0.087	-0.095	-0.142
BOEING	-0.096	-0.145	-0.151	-0.234	**MERCK**	-0.081	-0.121	-0.130	-0.195
CATERPILLAR	-0.065	-0.098	-0.111	-0.170	**MICROSOFT**	-0.048	-0.071	-0.082	-0.128
CHEVRON	-0.066	-0.099	-0.104	-0.162	**NIKE**	-0.230	-0.327	-0.374	-0.553
CISCO	-0.051	-0.074	-0.092	-0.134	**PROCTER & GAMBLE**	-0.059	-0.090	-0.101	-0.151
COCA-COLA	-0.038	-0.057	-0.061	-0.096	**SALESFORCE**	-0.145	-0.216	-0.243	-0.376
DISNEY	-0.071	-0.106	-0.115	-0.175	**TRAVELERS**	-0.079	-0.122	-0.130	-0.205
DOW	-0.081	-0.126	-0.127	-0.198	**UNITED HEALTH**	-0.081	-0.120	-0.139	-0.219
GOLDMAN SACHS	-0.074	-0.114	-0.120	-0.193	**VERIZON**	-0.055	-0.083	-0.086	-0.129
HOME DEPOT	-0.076	-0.112	-0.126	-0.192	**VISA**	-0.073	-0.111	-0.128	-0.201
HONEY WELL	-0.044	-0.071	-0.075	-0.124	**WALMART**	-0.053	-0.078	-0.084	-0.129

Panel 3.2: FHS Value-at-Risk with n. 1,000,000 independent random trials

Company Name	FHS Value-at-Risk				Company Name	FHS Value-at-Risk			
	95%		99%			95%		99%	
	10 days	22 days	10 days	22 days		10 days	22 days	10 days	22 days
3M	-0.0627	-0.0950	-0.1027	-0.1566	**IBM**	-0.0585	-0.0893	-0.0948	-0.1452
AMAZON	-0.0778	-0.1128	-0.1294	-0.1916	**INTEL**	-0.0912	-0.1367	-0.1497	-0.2242
AMERICAN EXPRESS	-0.0608	-0.0935	-0.0980	-0.1548	**JOHNSON & JOHNSON**	-0.0489	-0.0751	-0.0793	-0.1256
AMGEN	-0.0701	-0.1046	-0.1137	-0.1725	**JPMORGAN CHASE**	-0.0684	-0.1056	-0.1106	-0.1737
APPLE	-0.0841	-0.1248	-0.1383	-0.2121	**MCDONALD'S**	-0.0591	-0.0870	-0.0936	-0.1405
BOEING	-0.0970	-0.1439	-0.1538	-0.2314	**MERCK**	-0.0804	-0.1199	-0.1287	-0.1956
CATERPILLAR	-0.0658	-0.0964	-0.1101	-0.1656	**MICROSOFT**	-0.0478	-0.0708	-0.0826	-0.1279
CHEVRON	-0.0656	-0.0992	-0.1033	-0.1588	**NIKE**	-0.2283	-0.3307	-0.3714	-0.5458
CISCO	-0.0502	-0.0746	-0.0888	-0.1351	**PROCTER & GAMBLE**	-0.0586	-0.0902	-0.0990	-0.1527
COCA-COLA	-0.0382	-0.0576	-0.0625	-0.0957	**SALESFORCE**	-0.1430	-0.2132	-0.2396	-0.3642
DISNEY	-0.0699	-0.1057	-0.1142	-0.1746	**TRAVELERS**	-0.0792	-0.1207	-0.1302	-0.2017
DOW	-0.0809	-0.1263	-0.1256	-0.1991	**UNITED HEALTH**	-0.0808	-0.1215	-0.1394	-0.2166
GOLDMAN SACHS	-0.0753	-0.1147	-0.1215	-0.1891	**VERIZON**	-0.0545	-0.0821	-0.0857	-0.1310
HOME DEPOT	-0.0758	-0.1124	-0.1260	-0.1927	**VISA**	-0.0734	-0.1110	-0.1278	-0.2012
HONEY WELL	-0.0450	-0.0709	-0.0763	-0.1240	**WALMART**	-0.0530	-0.0781	-0.0858	-0.1285

Table 4.
Rates of return for the constituents of the DJIA index

The table contains the prices and the rates of return for the 30 constituents of the Dow Jones Industrial Average index, computed as percentage variations of the stock prices at 1 week and 1 month, from **June 28th, 2024.**

Company Name	Stock Price June 28th, 2024	Stock Price July 5th, 2024	Stock Price July 26th, 2024	Rates of return	
				1 week	1 month
3M	102.19	101.32	127.16	**-0.85%**	**24.43%**
AMAZON	193.25	200.00	182.50	**3.49%**	**-5.56%**
AMERICAN EXPRESS	231.55	235.63	245.89	**1.76%**	**6.19%**
AMGEN	312.45	310.88	334.85	**-0.50%**	**7.17%**
APPLE	210.62	226.34	217.96	**7.46%**	**3.48%**
BOEING	182.01	184.83	186.89	**1.55%**	**2.68%**
CATERPILLAR	333.10	328.35	350.48	**-1.43%**	**5.22%**
CHEVRON	156.42	154.31	157.84	**-1.35%**	**0.91%**
CISCO	47.51	46.65	47.88	**-1.81%**	**0.78%**
COCA-COLA	63.65	63.76	67.05	**0.17%**	**5.34%**
DISNEY	99.29	97.99	89.93	**-1.31%**	**-9.43%**
DOW	53.05	52.23	52.86	**-1.55%**	**-0.36%**
GOLDMAN SACHS	452.32	464.75	499.03	**2.75%**	**10.33%**
HOME DEPOT	344.24	334.58	359.51	**-2.81%**	**4.44%**
HONEY WELL	213.54	212.24	202.74	**-0.61%**	**-5.06%**
IBM	30.97	32.02	31.35	**3.39%**	**1.23%**
INTEL	172.95	176.02	191.75	**1.78%**	**10.87%**
JOHNSON & JOHNSON	146.16	146.48	160.64	**0.22%**	**9.91%**
JPMORGAN CHASE	202.26	204.79	212.24	**1.25%**	**4.93%**
MCDONALD'S	254.84	251.09	252.00	**-1.47%**	**-1.11%**
MERCK	123.80	126.45	125.26	**2.14%**	**1.18%**
MICROSOFT	446.95	467.56	425.27	**4.61%**	**-4.85%**
NIKE	75.37	75.43	72.56	**0.08%**	**-3.73%**
PROCTER & GAMBLE	17.46	17.23	18.03	**-1.32%**	**3.26%**
SALESFORCE	257.10	263.19	262.71	**2.37%**	**2.18%**
TRAVELERS	203.34	201.87	213.85	**-0.72%**	**5.17%**
UNITED HEALTH	509.26	488.01	569.72	**-4.17%**	**11.87%**
VERIZON	41.24	41.27	40.09	**0.07%**	**-2.79%**
VISA	262.47	270.36	259.46	**3.01%**	**-1.15%**
WALMART	67.71	70.04	69.78	**3.44%**	**3.06%**

Table 5.
Performance of the Asset Allocation (SEXTILES)

The table contains the average (**Panel 5.1**) and the cumulative (**Panel 5.2**) rates of return at 1 week and 1 month for the sextiles of the 30 constituents related to the Dow Jones Industrial Average index. The table reports the values for the Filtered Historical Simulation (**FHS**) Value-at-Risk at 95% and 99%, computed at 10 days and 22 days ahead. It considers n. 50,000 and n. 1,000,000 independent random trials. The first sextile of the constituents reports the average rates of return and the cumulative rates of return for the highest risk, i.e. the lowest value of the **FHS VaR,** assuming that an investor holds the formed portfolios for one week and one month period; whereas, the last sextile of the constituents reports the average rates of return and the cumulative rates of return for the lowest risk, i.e. the highest value of the **FHS VaR**, assuming that an investor holds the formed portfolios for one week and one month period.

Panel 5.1 Average Rates of Return (sextiles)

Sextiles of constituents	**Average Rates of Return**															
	FHS VaR 95%								**FHS VaR 99%**							
	50,000				**1,000,000**				**50,000**				**1,000,000**			
	10 days		**22 days**		**10 days**		**22 days**		**10 days**		**22 days**		**10 days**		**22 days**	
	1 week	**1 month**	**1 week**	**1 month**	**1 week**	**1 month**	**1 week**	**1 month**	**1 week**	**1 month**	**1 week**	**1 month**	**1 week**	**1 month**	**1 week**	**1 month**
1° sextile	2.65%	3.10%	0.85%	2.33%	2.65%	3.10%	0.85%	2.33%	2.65%	3.10%	0.32%	4.78%	0.32%	4.78%	0.32%	4.78%
2° sextile	-0.16%	-0.81%	1.49%	6.41%	-0.16%	2.46%	1.49%	6.41%	0.75%	2.30%	2.07%	1.67%	3.08%	0.62%	2.07%	1.67%
3° sextile	0.23%	2.27%	0.38%	-0.91%	0.23%	2.27%	0.73%	-1.35%	-0.68%	2.43%	0.32%	1.39%	-0.68%	2.43%	0.68%	0.94%
4° sextile	-0.12%	8.34%	-0.12%	8.34%	-0.12%	8.34%	-0.47%	8.78%	-0.74%	7.75%	-0.12%	8.34%	-0.74%	7.75%	-0.47%	8.78%
5° sextile	0.82%	0.73%	0.82%	0.73%	0.82%	0.73%	0.82%	0.73%	0.39%	0.86%	-0.23%	0.27%	1.06%	2.03%	-0.23%	0.27%
6° sextile	0.52%	1.22%	0.52%	1.22%	0.52%	1.22%	0.52%	1.22%	1.57%	1.68%	1.57%	1.68%	0.89%	0.51%	1.57%	1.68%

Panel 5.2: Cumulative Rates of Return (sextiles)

Sextiles of constituents	Cumulative rates of return															
	FHS VaR 95%								FHS VaR 99%							
	50,000				1,000,000				50,000				1,000,000			
	10 days		22 days		10 days		22 days		10 days		22 days		10 days		22 days	
	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month
1° sextile	13.24%	15.49%	4.23%	11.65%	13.24%	15.49%	4.23%	11.65%	13.24%	15.49%	1.60%	23.88%	1.60%	23.88%	1.60%	23.88%
2° sextile	-0.81%	12.30%	7.46%	32.03%	-0.81%	12.30%	7.46%	32.03%	3.74%	11.51%	10.34%	8.33%	15.38%	3.12%	10.34%	8.33%
3° sextile	1.14%	11.36%	1.88%	-4.53%	1.14%	11.36%	3.63%	-6.77%	-3.42%	12.15%	1.62%	6.94%	-3.42%	12.15%	3.38%	4.71%
4° sextile	-0.61%	41.69%	-0.61%	41.69%	-0.61%	41.69%	-2.37%	43.92%	-3.69%	38.76%	-0.61%	41.69%	-3.69%	38.76%	-2.37%	43.92%
5° sextile	4.12%	3.65%	4.12%	3.65%	4.12%	3.65%	4.12%	3.65%	1.94%	4.30%	-1.14%	1.37%	5.31%	10.14%	-1.14%	1.37%
6° sextile	2.58%	6.12%	2.58%	6.12%	2.58%	6.12%	2.58%	6.12%	7.84%	8.40%	7.84%	8.40%	4.47%	2.55%	7.84%	8.40%

Table 6.
Performance of the Asset Allocation (TERTILES)

The table contains the average (**Panel 6.1**) and the cumulative (**Panel 6.2**) rates of return at 1 week and 1 month for the tertiles of the 30 constituents related to the Dow Jones Industrial Average index. The table reports the values for the Filtered Historical Simulation (**FHS**) Value-at-Risk at 95% and 99%, computed at 10 days and 22 days ahead. It considers n. 50,000 and n. 1,000,000 independent random trials. The first tertile of the constituents reports the average rates of return and the cumulative rates of return for the highest risk, i.e. the lowest value of the **FHS VaR**, assuming that an investor holds the formed portfolios for one week and one month period; whereas, the last tertile of the constituents reports the average rates of return and the cumulative rates of return for the lowest risk, i.e. the highest value of the **FHS VaR**, assuming that an investor holds the formed portfolios for one week and one month period.

Panel 6.1 Average Rates of Return (tertiles)

Tertiles of constituents	Average rates of return															
	FHS VaR 95%								FHS VaR 99%							
	50,000				1,000,000				50,000				1,000,000			
	10 days		22 days		10 days		22 days		10 days		22 days		10 days		22 days	
	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month
1° tertile	1.24%	2.78%	1.17%	4.37%	1.24%	2.78%	1.17%	4.37%	1.70%	2.70%	1.19%	3.22%	1.70%	2.70%	1.19%	3.22%
2° tertile	0.05%	5.30%	0.13%	3.72%	0.05%	5.30%	0.13%	3.72%	-0.71%	5.09%	0.10%	4.86%	-0.71%	5.09%	0.10%	4.86%
3° tertile	0.67%	0.98%	0.67%	0.98%	0.67%	0.98%	0.67%	0.98%	0.98%	1.27%	0.67%	0.98%	0.98%	1.27%	0.67%	0.98%

Panel 6.2: Cumulative Rates of Return (tertiles)

Tertiles of constituents	Cumulative rates of return															
	FHS VaR 95%								FHS VaR 99%							
	50,000				1,000,000				50,000				1,000,000			
	10 days		22 days		10 days		22 days		10 days		22 days		10 days		22 days	
	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month	1 week	1 month
1° tertile	12.43%	27.79%	11.68%	43.68%	12.43%	27.79%	11.68%	43.68%	16.98%	27.00%	11.94%	32.21%	16.98%	27.00%	11.94%	32.21%
2° tertile	0.52%	53.05%	1.27%	37.16%	0.52%	53.05%	1.27%	37.16%	-7.11%	50.91%	1.01%	48.63%	-7.11%	50.91%	1.01%	48.63%
3° tertile	6.70%	9.76%	6.70%	9.76%	6.70%	9.76%	6.70%	9.76%	9.78%	12.69%	6.70%	9.76%	9.78%	12.69%	6.70%	9.76%